

Chaotic inflation on the Randall-Sundrum 2-brane model

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Abstract

We construct an inflation model on the Randall-Sundrum I (RSI) brane where a bulk scalar field stabilizes the inter-brane separation. We study impact of the bulk scalar field on the inflationary dynamics on the brane. We proceed in two different approaches: in the first approach, the stabilizing field potential is directly appeared in the Friedmann equation and the resulting scenario is effectively a two-field inflation. In the second approach the stabilization mechanism is considered in the context of a warp factor so that there is just one field present that plays the roles of both inflaton and stabilizer. We study constraints imposed on the model parameters from recent observations.

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1 Introduction

The idea of braneworld models is an exciting way to solve the problems facing standard cosmology and particle physics. The basic idea is that the standard model matter is confined to a 3-brane while gravity propagates in the higher dimensional bulk. This means that gravity is fundamentally a higher dimensional interaction and we only see the effective $4D$ theory on the brane. One of the first suggestions in this respect was the model proposed by Arkani-Hamed *et al* [1]. In this model the difference between the fundamental scales of gravity and electroweak is due to the existence of large extra dimensions that are accessible only for graviton and possibly non-standard matter. Later, a different setup was proposed by Randall and Sundrum [2]. Their model has two Minkowski branes and a single extra dimension. The branes have equal and opposite tensions and we live in the negative tension brane. The hierarchy problem could be solved in this setup by the exponentially changing metric along the extra dimension.

It has been shown firstly by Binetroy *et al* [3] that the Randall-Sundrum 2-brane model in the absence of any source (including a cosmological constant) in the bulk, leads to a phenomenologically unacceptable cosmological evolution. However, when there is a cosmological constant in the bulk, the usual Friedmann equations are recovered at low energy but in this case there is a strict constrain between the energy density of matter on the branes which is undesirable phenomenologically [4]. Later, it has been shown that the correct cosmological evolution equations are recovered if one stabilizes the separation between the two branes [5]. To do this end, one should introduce a massive scalar field called Radion to stabilize the inter-brane separation. The simplest mechanism for radius stabilization has been proposed by Goldberger and wise (GW) [6]. They introduced a bulk scalar field with different vacuum expectation values (VEVs) on the two branes. After the stabilization, there is no constraint on the matter density on the branes. Cosmological dynamics and phenomenology of the model are crucially dependant on the mass and coupling of the Radion field. One could achieve the desired result if the mass of the Radion is of the order of TeV and its couplings to Standard Model (SM) fields is $O(TeV^{-1})$. It has been shown in reference [7] that in the GW mechanism, these conditions are indeed satisfied. Reference [8] provides a thorough analysis of the Radion dynamics and RS I cosmology.

One faces the issue of Radion stabilization only in the RS I and ADD solutions. After proposing the 2-brane model, Randall and Sundrum introduced a new scenario [9] (RS II), in which the extra dimension is infinite in size, which effectively means moving the negative tension brane of RS I scenario to infinity. There is only a single (positive tension) brane and no Radion is needed; its wave function diverges away from the brane. Because of this simplicity, much of the early works on brane cosmology have focused on the RS II solution (see [10] and references therein).

In recent years, cosmologists have shown renewed interest in studying the cosmological implications of the RS I model [11]. The reason for this is that one can include the quantum gravity effects more readily in this scenario: Kaluza-Klein excitations of the bulk graviton modes are supposed to be equivalent to bound states of a strongly coupled, nearly conformal field theory residing on the TeV brane [12]. At temperatures above the TeV scale, the TeV

brane is assumed hidden behind a horizon [13] which is formed by a black hole in the bulk [14]. In fact, at high temperatures there are two possible scenarios. The first is that the TeV brane simply does not exist at early times; the horizon shields the region where the TeV brane might exist. As the temperature drops and the horizon recedes, it eventually uncloaks the TeV brane, and the true SM degrees of freedom emerge. An alternative possibility with similar physical consequences is that a TeV brane exists at the scale associated with the temperature of the theory. Only when the temperature drops to appropriately low scale will the brane settle at its true minimum, analogously to the behavior of other moduli in the early universe. It has been noted that the emergence of the TeV brane from the horizon may occur around the same time as the electroweak phase transition [15]. Ref. [16] provided an alternative picture of a first order phase transition which leads to the appearance of the TeV brane at this epoch.

Although standard Big Bang cosmology has been very successful in explaining the evolution of the universe as we see today; but despite all of its successes there exist some problems unsolved in this framework. The most notable of these are the flatness problem, the observed low density of monopoles and the horizon problem [17,18]. Theories with inflation (a period in the early universe characterized by an exponential expansion of the scale factor) offer solutions to these problems and in fact are the only ones to do so. Inflation, however, suffers from its own set of problems. Primary amongst these, is the generation of a correct (scalar) potential that would drive inflation [19], and equally important the mechanism of a non-contrived exit of the universe from an inflationary phase. This graceful exit problem has been plaguing both cosmologists and string theorists with no simple solution in sight. Extension of the inflation paradigm to braneworld scenarios have provided a variety of new ideas in the spirit of the particle physics and cosmology.

With these preliminaries, in this paper, we investigate the chaotic inflation scenario in the Randall-Sundrum 2-brane (RS I) model. We assume that the size of the extra dimension is stabilized using appropriate Radion potential. The modified Friedmann equations for the branes are used to obtain the evolution of the universe in the inflation era. We adopt two relatively different approaches in which in the first approach, the stabilizing field potential directly appears in the Friedmann equation and the resulting scenario is effectively a two-field inflation. In the second approach, the stabilization mechanism is considered in the context of a warp factor so that there is just one field present that plays the roles of both inflaton and stabilizer. These two approaches are compared and by analyzing the parameter space of the model we investigate possible realization of the graceful exit in this setup. We study constraints imposed on the model parameters from recent observations.

2 The model

Consider a RS model with two branes located at $y = 0$ and $y = 1/2$. We suppose that the metric has the form

$$\begin{aligned} ds^2 &= n^2(y, t)dt^2 - a^2(y, t)(dx_1^2 + dx_2^2 + dx_3^2) - b^2(y, t)dy^2, \\ &\equiv \tilde{g}_{AB}(x, y)dx^A dx^B. \end{aligned} \tag{1}$$

where y is the coordinate of the extra dimension and $A, B = 0, 1, 2, 3, 5$. Components of the Einstein tensor for this metric are [3]

$$\begin{aligned}
G_{00} &= 3 \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{\dot{a}\dot{b}}{ab} - \frac{n^2}{b^2} \left(\frac{a''}{a} + \left(\frac{a'}{a} \right)^2 - \frac{a'b'}{ab} \right) \right], \\
G_{ii} &= \frac{a^2}{b^2} \left[\left(\frac{a'}{a} \right)^2 + 2 \frac{a' n'}{a n} - \frac{b' n'}{b n} - 2 \frac{b' a'}{b a} + 2 \frac{a''}{a} + \frac{n''}{n} \right] + \frac{a^2}{n^2} \left[- \left(\frac{\dot{a}}{a} \right)^2 + 2 \frac{\dot{a} \dot{n}}{a n} - 2 \frac{\ddot{a}}{a} + \right. \\
&\quad \left. \frac{\dot{b}}{b} \left(-2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\ddot{b}}{b} \right], \\
G_{05} &= 3 \left[\frac{n' \dot{a}}{n a} + \frac{a' \dot{b}}{ab} - \frac{\dot{a}'}{a} \right], \\
G_{55} &= 3 \left[\frac{a'}{a} \left(\frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{n^2} \left(\frac{\dot{a}}{a} \left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right) \right], \tag{2}
\end{aligned}$$

where a prime denotes differentiation with respect to y while a dot marks differentiation with respect to t , the cosmic time. We assume that there exists a cosmological constant in the bulk so the overall energy-momentum tensor has two parts. One from the bulk cosmological constant of the form

$$T_{ab}^{(bulk)} = \tilde{g}_{ab} \Lambda, \tag{3}$$

and the other from the matter on the branes which has the following form

$$\begin{aligned}
T_a^b \text{ (brane)} &= \frac{\delta(y)}{b} \text{diag} \left(V_* + \rho_*, V_* - p_*, V_* - p_*, V_* - p_*, 0 \right) + \\
&\quad \frac{\delta(y - \frac{1}{2})}{b} \text{diag} \left(-V + \rho, -V - p, -V - p, -V - p, 0 \right), \tag{4}
\end{aligned}$$

where V_* is the (positive) tension (or equivalently the cosmological constant) of the brane located at $y = 0$, ρ_* and p_* are the density and pressure of the matter situated on the positive tension brane (with an equation of state of the form $p_* = w_* \rho_*$) and ρ and p are the density and pressure of the matter on the negative tension brane (the brane which we live in). Both sets of densities and pressures are measured with respect to \tilde{g} . In the absence of any energy-momentum sources on the branes, that is, where $\rho, p, \rho_*, p_* \rightarrow 0$, one recovers the static Randall-Sundrum solution of the form

$$n(y) = a(y) = e^{-|y|m_0 b_0}, \tag{5}$$

where the relations between Λ, V_*, V and m_0 are given by

$$\begin{aligned}
V_* &= \frac{6m_0}{\kappa^2} = -V, \\
\Lambda &= -\frac{6m_0^2}{\kappa^2}. \tag{6}
\end{aligned}$$

The effective 4D Planck scale is then given by

$$(8\pi G_N)^{-1} = M_{Pl}^2 \equiv \frac{1 - \Omega_0^2}{\kappa^2 m_0}, \quad (7)$$

where Ω_0 is the present-day value of the warp factor and is given by

$$\Omega_0 \equiv e^{-m_0 b_0/2} \quad (8)$$

The warp factor in general is b -dependent so we define

$$\Omega_b \equiv e^{-m_0 b(t)|y|/2} \quad (9)$$

where we measure it at $y = \frac{1}{2}$, at our brane. When $b = b_0 = \text{constant}$, then $\Omega_b = \Omega_0$.

Now we include the Radion in our equations. Without stabilization, the Radion has no mass, so only its potential enters into the action. We assume that the coefficients in the metric (1) have the following forms

$$\begin{aligned} a(y, t) &= a(t)\Omega(y, b(t)) [1 + \delta\bar{a}(y, t)] \\ n(y, t) &= \Omega(y, b(t)) [1 + \delta\bar{n}(t, y)] \\ b(t, y) &= b(t)[1 + \delta\bar{b}(y, t)]. \end{aligned} \quad (10)$$

The modified Friedmann equation for the geometry given by the metric (1) is [5]

$$\frac{\dot{a}^2}{a^2} + (m_0 b) \frac{\Omega_b^2}{1 - \Omega_b^2} \frac{\dot{a} \dot{b}}{a b} - \frac{(m_0 b)^2}{4} \frac{\Omega_b^2}{1 - \Omega_b^2} \frac{\dot{b}^2}{b^2} = \frac{\kappa^2 m_0}{3} \frac{1}{1 - \Omega_b^2} (\rho_* + \rho \Omega_b^4 + W_r(b)) + \epsilon^2 \quad (11)$$

where $\epsilon^2 = O((\delta\bar{a})^2, (\delta\bar{n})^2, (\delta\bar{b})^2)$ and $W_r(b)$ is the Radion potential. From the above Friedmann equation one can deduce that unless there is a strict constraint between the matter densities on the two branes, the universe evolves in a very unconventional manner which is not supported by present day observations [3,5]. Also it is very undesirable that the matter densities on the two branes should obey such a constraints.

In order to avoid the above mentioned problems, we should stabilize the radius. This can be done by assuming that the Radion is massive. An elegant mechanism for Radion stabilization has been proposed by Goldberger and Wise. After the stabilization, the Friedmann equations are [5]

$$\begin{aligned} \frac{\ddot{a}^2}{a^2} &= \frac{8\pi G_N}{3} \left(f^4(b)(\rho_* + \rho_{vis}) + \frac{1}{4} \frac{3}{8\pi G_N} (m_0 b)^2 \left(\frac{\Omega_b}{1 - \Omega_b^2} \right)^2 \frac{\dot{b}^2}{b^2} + \overline{W}_r(b) \right) \\ &= \frac{8\pi G_N}{3} \left(\frac{1}{2} \dot{\psi}^2 + \overline{W}_r(\psi) + f^4(b)(\rho_* + \rho_{vis}) \right), \end{aligned} \quad (12)$$

and

$$\frac{\ddot{a}}{a} = -4\pi G_N \left(f^4(b) [(\rho_* + \rho_{vis}) + 3(p_* + p_{vis})] + 2(\dot{\psi}^2 - \overline{W}_r(\psi)) \right). \quad (13)$$

Here ψ is the canonically normalized Radion and we performed a conformal transformation of the metric as

$$\begin{aligned} a(t) &= f(b(\bar{t})) \bar{a}(\bar{t}) \\ dt &= f(b(\bar{t})) d\bar{t}, \end{aligned} \quad (14)$$

where the function $f(b)$ is defined by

$$f(b) = \left(\frac{1 - \Omega_0^2}{1 - \Omega_b^2} \right)^{1/2}. \quad (15)$$

With the above definitions, the Radion potential in the absence of any source is

$$\bar{W}_r(b) = f^4(b) W_r(b), \quad (16)$$

and the canonically normalized Radion is given by

$$m_0 b(t) = \sqrt{\frac{2}{3}} \frac{\psi(t)}{\Omega_0 M_{pl}} (1 - \Omega_0^2) \sim \sqrt{\frac{2}{3}} \frac{\psi(t)}{\Omega_0 M_{pl}}. \quad (17)$$

The Friedmann equations (12) and (13) are derived by neglecting the back-reaction of the Radion mass on the background metric and the wave function of the Radion. However, it has been shown in reference [8] that including these considerations will not change the results. It should also be noted that equations (12) and (13) are represented in the Einstein frame so in the low energy limit with a stabilized Radion ($\dot{b} = 0$), the universe evolves as in usual cosmology in this frame. With the definition (14), it is obvious that up to a small fluctuation, the evolution of the universe is also the same in the original frame.

3 The Goldberger-Wise Stabilization Mechanism

The model for Radion stabilization which we use here is the one proposed by Goldberger and Wise [6]. In their model, a bulk scalar field which also contains two potentials on TeV and Planck branes, plays the role of stabilizer. The whole action for this model is

$$S = \int d^5x \sqrt{-\tilde{G}} \left(-\frac{1}{2\kappa_5^2} R - \Lambda + \tilde{G}^{AB} \delta_A \Phi \delta_B \Phi - U_{bulk}(\Phi) \right) + S_{TeV} + S_{Pl}, \quad (18)$$

where the action on the branes are given by

$$\begin{aligned} S_{Pl} &= \int \sqrt{-g} d^4x (L_{m0} - U_0(\Phi)) \\ S_{TeV} &= \int \sqrt{-g} d^4x (L_{m1/2} - U_{1/2}(\Phi)). \end{aligned} \quad (19)$$

Here \tilde{G}_{AB} is the 5D metric given by equation (1) and the potentials in the bulk and in the brane are given by

$$\begin{aligned} U_{bulk}(\Phi) &= \frac{1}{2} m_s^2 \Phi^2 \\ U_i(\Phi) &= \lambda_i (\Phi^2 - v_i^2)^2 \end{aligned} \quad (20)$$

where $i = 0, \frac{1}{2}$ and v_i 's have the dimension of $(mass)^{3/2}$ and could be different on the two branes. With this action, Goldberger and Wise find the effective 4D potential for the Radion as

$$W_r(b) = 4m_0 e^{-2m_0 b} \left(v_{1/2} - v_0 e^{-\epsilon m_0 b/2} \right)^2 \left(1 + \frac{\epsilon}{4} \right) - \epsilon m_0 v_{1/2} e^{-(4+\epsilon)m_0 b/2} \left(2v_{1/2} - v_0 e^{-\epsilon m_0 b/2} \right) \quad (21)$$

where ϵ is defined by

$$\epsilon = \frac{4m_s^2}{m_0}. \quad (22)$$

Substituting from equation (17) for the canonically normalized Radion, we get

$$W_r(\psi) = 4m_0 e^{-2(\sqrt{\frac{2}{3}} \frac{\psi}{\Omega_0 M_{pl}})} \left(v_{1/2} - v_0 e^{-\epsilon(\sqrt{\frac{2}{3}} \frac{\psi}{\Omega_0 M_{pl}})/2} \right)^2 \left(1 + \frac{\epsilon}{4} \right) - \epsilon m_0 v_{1/2} e^{-(4+\epsilon)(\sqrt{\frac{2}{3}} \frac{\psi}{\Omega_0 M_{pl}})/2} \left(2v_{1/2} - v_0 e^{-\epsilon(\sqrt{\frac{2}{3}} \frac{\psi}{\Omega_0 M_{pl}})/2} \right). \quad (23)$$

Figure 1 shows the shape of the Golberger-Wise stabilization potential in our setup as given by equation (23).

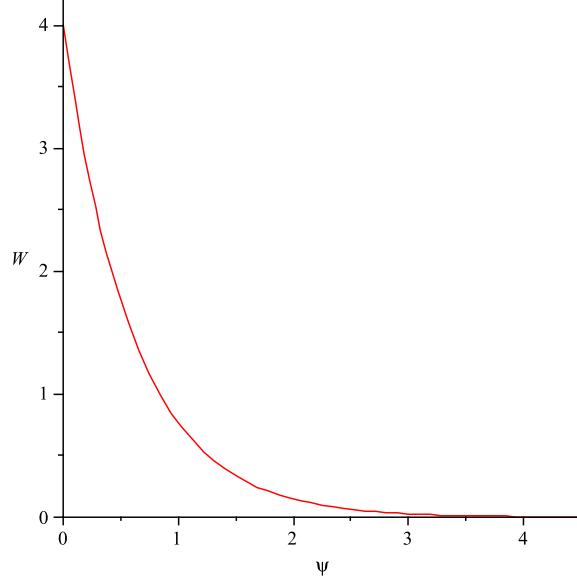


Figure 1: Stabilization potential in Goldberger-Wise mechanism as given by equation (23).

4 Chaotic inflation in the RS I model

Now we have the tools to investigate the inflation in the Randall-Sundrum 2-brane scenario. Our starting points are the modified Friedmann equations (12) and (13). We assume that the inflation takes place in the TeV brane so we set $\rho_* = 0$.

We assume also that there is an inflaton field ϕ , with a potential $V(\phi)$ on the TeV brane where its energy density and pressure are given as

$$\rho = \frac{1}{2}\dot{\phi}^2 + V \quad (24)$$

and

$$p = \frac{1}{2}\dot{\phi}^2 - V \quad (25)$$

respectively. To have an inflationary era, there should be a phase of accelerated expansion in the early stages of the universe evolution in which $\ddot{a} > 0$. From equation (13) this means that

$$f^4(b)(\rho_{vis} + 3p_{vis}) + 2(\dot{\psi}^2 - \overline{W}_r(\psi)) < 0 \quad (26)$$

Substituting (24) and (25) into (26) we get

$$f^4(b)[2(\dot{\phi}^2 - V)] + 2(\dot{\psi}^2 - \overline{W}_r(\psi)) < 0, \quad (27)$$

which is the condition for realization of inflation in our setup. The Klein-Gordon equation governing the evolution of the inflaton field is as usual

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (28)$$

In the standard Friedmann cosmology, the energy condition required for realization of inflation yields $\frac{1}{2}\dot{\phi}^2 < V(\phi)$ which means that the potential energy of the inflaton dominates over its kinetic energy. Imposing the slow-roll approximation *i.e.*

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi) \quad (29)$$

and

$$\ddot{\phi} \ll 3H\dot{\phi}, \quad (30)$$

on our model, the Hubble parameter for a model universe where the only matter is an inflaton field on the TeV brane, is given by equation (12) as

$$H^2 = \frac{8\pi G_N}{3} \left[\frac{1}{2}\dot{\phi}^2 + \overline{W}_r(\psi) + f^4(b)\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \right]. \quad (31)$$

In the slow-roll approximation this becomes

$$H^2 \simeq \frac{8\pi G_N}{3} \left[\frac{1}{2}\dot{\psi}^2 + \overline{W}_r(\psi) + f^4(b)(V(\phi)) \right]. \quad (32)$$

Also from Klein-Gordon equation one finds

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H}. \quad (33)$$

We assume that potential energy of the Radion field dominates its kinetic energy so that the first term on the right hand side of equation (32) can be neglected. Now, the slow-roll parameters are defined as

$$\begin{aligned}\varepsilon &\equiv \frac{M_{pl}^2}{16\pi} \left(\frac{W' + fV'}{W + fV} \right)^2 \\ \eta &\equiv \frac{M_{pl}^2}{16\pi} \left(\frac{W'' + fV''}{W + fV} \right),\end{aligned}\quad (34)$$

where a prime denotes differentiation with respect to the argument in each case. We note that these definitions of the slow-roll parameters are very similar to the two-field inflation. So, inflation in our setup is effectively a two-field inflation. In other words, incorporation of the role played by Radion in this inflation scenario turns it to an effective two-field inflation. The slow-roll approximations are valid when

$$\begin{aligned}\varepsilon &\ll 1 \\ |\eta| &\ll 1.\end{aligned}\quad (35)$$

The inflationary phase ends when ε and $|\eta|$ grow of order unity. The amount of inflation is described by the number of e-folds which is defined as

$$N \equiv \ln\left(\frac{a_f}{a_i}\right) = \int_{t_i}^{t_f} H dt. \quad (36)$$

Substituting for H from equation (32) we find

$$N = \int_{\psi_i}^{\psi_f} \int_{\phi_i}^{\phi_f} \left[\frac{8\pi G_N}{3} \left(\frac{1}{2} \dot{\psi}^2 + \overline{W}_r(\psi) + f^4(b) V(\phi) \right) \right]^{1/2} d\phi d\psi. \quad (37)$$

It is obvious that the number of e-folds in our setup is larger than the standard case due to incorporation of the Radion potential which is positive as figure 1 shows. Figures 2 and 3 show the graceful exit from the inflationary phase. The condition for graceful exit is $\varepsilon = 1$, however it is possible essentially to fulfill the condition $|\eta| = 1$ before fulfilling $\varepsilon = 1$ and in this case inflation terminates when the condition $|\eta| = 1$ is fulfilled.

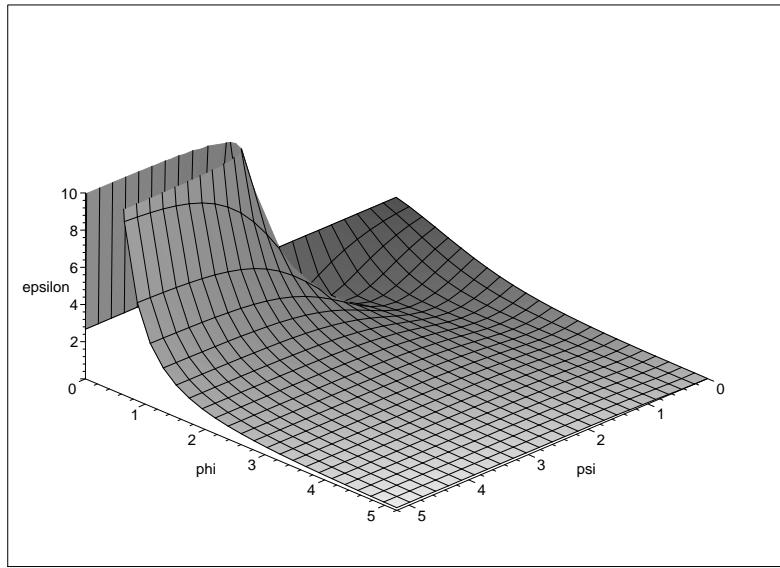


Figure 2: Graceful exit from inflationary phase. ϵ versus ϕ/M_{pl} and ψ/M_{pl}

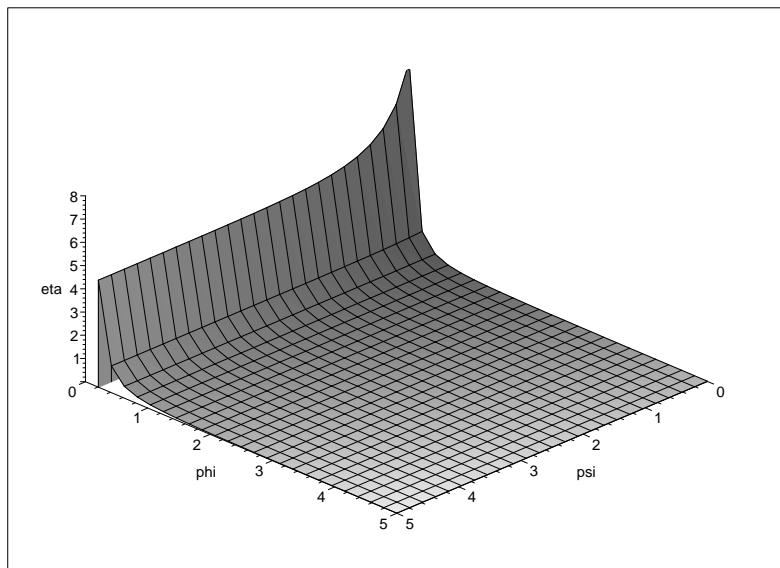


Figure 3: Graceful exit from inflationary phase. η versus ϕ/M_{pl} and ψ/M_{pl} .

5 Perturbations

Density perturbations due to quantum mechanical fluctuations of the inflaton field, have important observational consequences for the present epoch of the universe because, it is believed that these perturbations are the source of large scale structure formation in the universe.

The fluctuations of the inflaton field are related to the curvature perturbations on comoving hypersurfaces by the following relation

$$R = \frac{H\delta\phi}{\dot{\phi}} \quad (38)$$

In the slow-roll limit the field fluctuation at the horizon crossing (or Hubble radius) is given by

$$\delta\phi = \frac{H}{2\pi} \quad (39)$$

This result is independent of the geometry and holds for a massless scalar field in de Sitter spacetime independently of the gravitational field equations. The amplitude of scalar perturbations is related to the density perturbation by the following relation

$$A_s^2 = 4 \frac{R^2}{25} \quad (40)$$

In our model, using equations (31),(32) and (38), this gives

$$A_s^2 \simeq \frac{2048}{675} \left(\frac{\pi(G_N)^3}{V^2} \right) \left(\frac{1}{2}\dot{\psi}^2 + \overline{W}_r(\psi) + f^4(b)V(\phi) \right)^3, \quad (41)$$

which is calculated at the Hubble crossing. One of the important parameters which is useful to constraint the inflationary models with observations is the spectral index, n_s . In the slow-roll limit, n_s is given by

$$n_s - 1 \equiv \frac{d \ln A_s^2}{d \ln k} = -6\varepsilon + 2\eta \quad (42)$$

where k is the comoving wavenumber. Recent WMAP five year result [20] combined with SDSS and SNIa data shows that $n_s \simeq 0.960$. We plot the value of n_s versus ϕ and ψ fields at the end of inflation in figure 4. Figure 5 shows the values of ϕ and ψ to match the observed value of n_s by WMAP. We use this figure to extract the values of ϕ_{end} and ψ_{end} and then using equation (37) we find that the number of e-folds in our model is $N \simeq 65 - 105$ which is slightly larger than the standard case which is expected as has been discussed after equation (37). It is also obvious from figure 5 that with suitable value of ψ the value of inflaton field ϕ could be below the planck scale thus avoiding the so called η -problem.

The amplitude of tensorial perturbations at the Hubble crossing is

$$A_T^2 = \frac{4}{25\pi} \left(\frac{H}{M_{pl}} \right)^4. \quad (43)$$

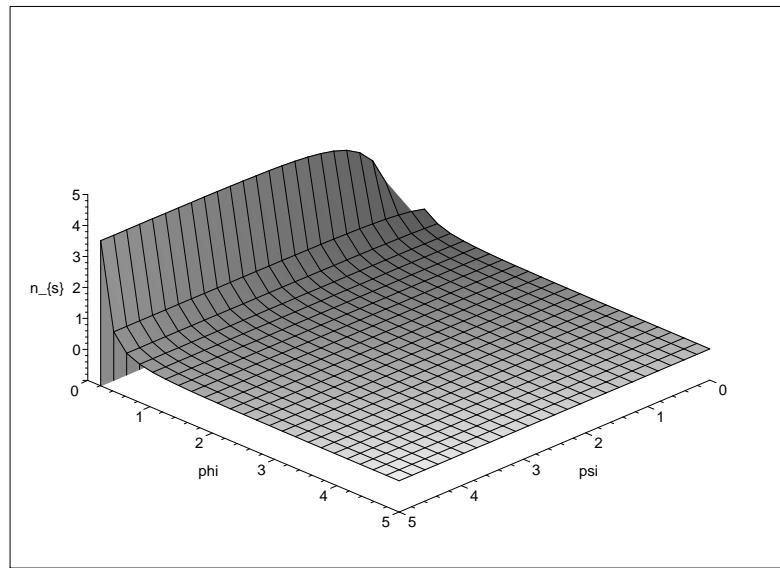


Figure 4: Value of n_s for different values of ϕ and ψ fields.

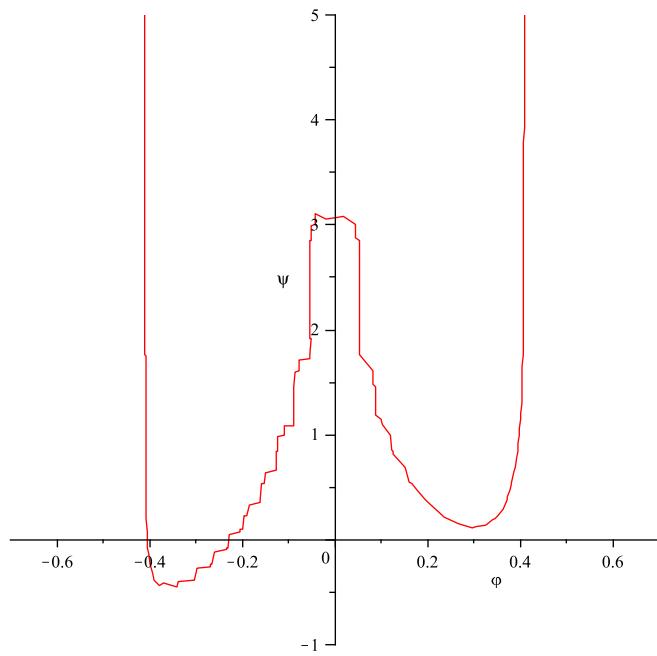


Figure 5: Acceptable values of ϕ and ψ to match the observed value of n_s by combined WMAP5+SDSS+SNIa dataset.

In the slow-roll approximation using equations (31) this gives

$$A_T^2 \simeq \frac{32\pi(G_N)^2}{75} \left(\frac{1}{2}\dot{\psi}^2 + \overline{W}_r(\psi) + f^4(b)V(\phi) \right). \quad (44)$$

Figures 6 and 7 shows the amplitude of scalar and tensorial perturbations for our model. These figures show that our model is favored by the recent observations from combined WMAP5+SDSS+SNIa dataset. The tensorial spectral index is given by

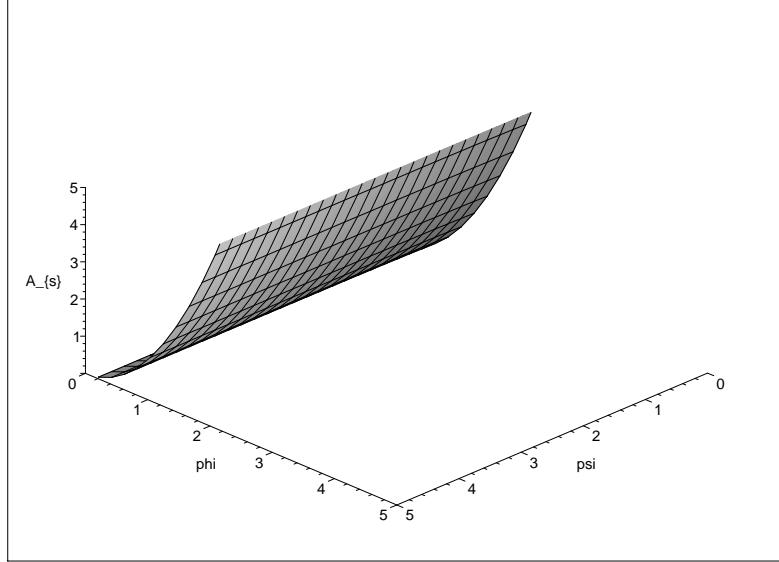


Figure 6: Amplitude of scalar perturbations vs ϕ and ψ fields .

$$n_T \equiv \frac{d \ln A_T^2}{d \ln k} = -2\varepsilon. \quad (45)$$

The relative amplitude of tensorial and scalar spectrums is therefore

$$\frac{A_T^2}{A_R^2} = 0.140625 \frac{V'^2}{G_N} \left(\frac{1}{2}\dot{\psi}^2 + \overline{W}_r(\psi) + f^4(b)(V(\phi)) \right)^{-2}. \quad (46)$$

Figure 8 shows the relative amplitude of tensorial and scalar perturbations for our model

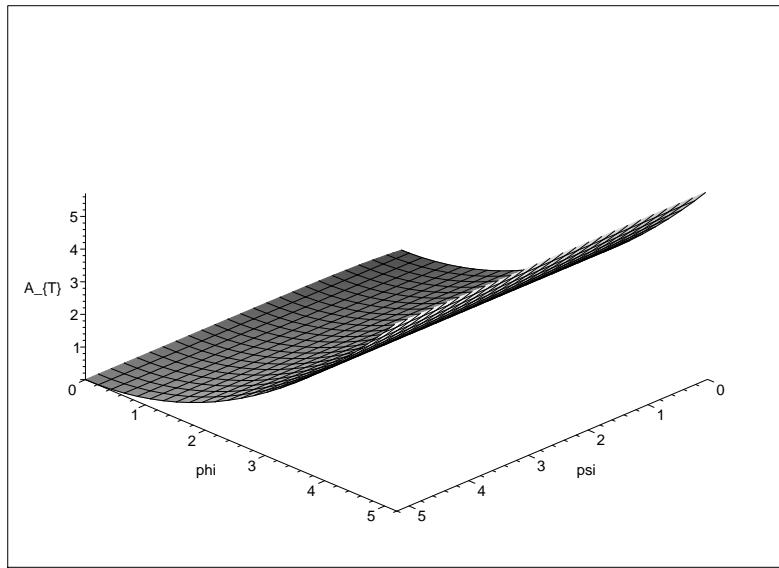


Figure 7: Amplitude of tensorial perturbations vs ϕ and ψ fields

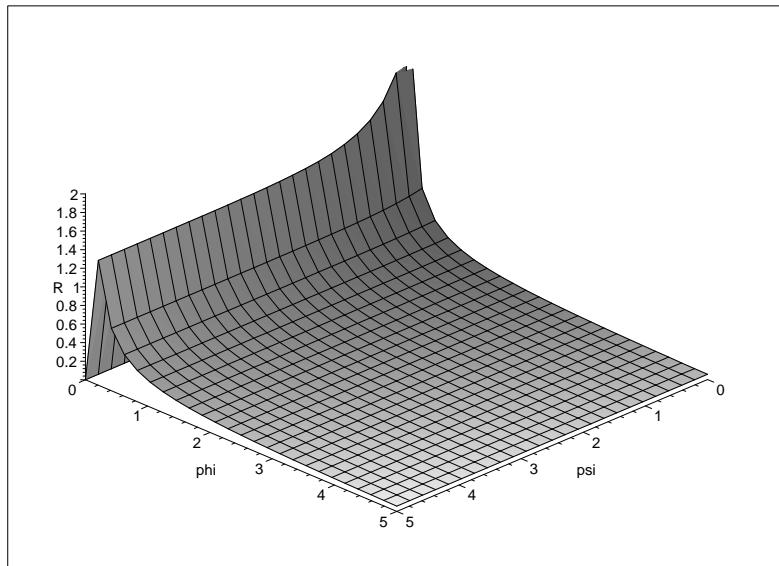


Figure 8: Relative Amplitude of tensorial and scalar perturbations vs ϕ and ψ fields

6 An alternative approach

An alternative representation of the Randall-Sundrum model mentioned in section 2, was presented by Cline and Firouzjahi [21]. Using action (18) for a RS I model with radion stabilization, they made a perturbative expansion of the coefficients of the metric as

$$\begin{aligned} n(t, y) &= n_0(y) + \delta n(t, y) & a(t, y) &= a_0(y) + \delta a(t, y) \\ b(t, y) &= b_0 + \delta b(t, y) & , \quad \Phi(t, y) &= n_0(y) + \delta \Phi(t, y) \end{aligned} \quad (47)$$

The Klein-Gordon equation for the scalar field, Φ , for this action is

$$\delta_t \left(\frac{1}{n} b a^3 \dot{\Phi} \right) - \delta_y \left(\frac{1}{b} a^3 n \Phi' \right) + b a^3 n \left[U' + U'_0 \delta(b y) + U'_{1/2} \delta(b(y - 1/2)) \right] = 0 \quad (48)$$

Inserting the metric in the form of (1) in Einstein equations with stress-energy tensor as (4), the solutions in zeroth order in perturbations are

$$\Phi_0(y) \simeq v_0 e^{-\epsilon m_0 b_0 y/2}; \quad a_0(y) \simeq m_0 b_0 y + \frac{\kappa^2}{12} v_0^2 (e^{-\epsilon m_0 b_0 y/2} - 1) \quad (49)$$

Where ϵ and v_0 are given in section 3. The Friedmann equations are

$$\begin{aligned} \left(\frac{\dot{a}_0}{a_0} \right)^2 &= \frac{8\pi G}{3} \left(\rho_* + \Omega^4 \rho \right) \\ \left(\frac{\dot{a}_0}{a_0} \right)^2 - \frac{\ddot{a}_0}{a_0} &= 4\pi G \left(\rho_* + p_* + \Omega^4 (\rho + p) \right). \end{aligned} \quad (50)$$

Here the warp factor, Ω , is defined as

$$\Omega \equiv e^{-a_0(1/2)} \quad (51)$$

where $a_0(y)$ is given by (49). This equation is valid before the stabilization when $b = b_0$ but in the inflation era, b_0 in the equation (49) should be replaced by b which is a function of time, so we have

$$\Omega(\Phi) = \exp \left[\frac{8}{m_0} \ln \frac{\Phi}{v_0} + \frac{\kappa^2}{3} v_0^2 \left(e^{-\epsilon \frac{1}{m_0} \ln \frac{\Phi}{m_0}} - 1 \right) \right]. \quad (52)$$

Now we turn our attention to inflation and we assume again that the only matter existed in the universe is an inflaton field whose energy density and pressure are given by (24) and (25) respectively. We note that the inflaton field plays the role of stabilizer in this setup. Substituting (24) and (49) into Friedmann equation (50), we get

$$H^2 = \frac{8\pi G}{3} \left\{ \exp \left[\frac{8}{m_0} \ln \frac{\Phi}{v_0} + \frac{\kappa^2}{3} v_0^2 \left(e^{-\epsilon \frac{1}{m_0} \ln \frac{\Phi}{m_0}} - 1 \right) \right] \left(\frac{1}{2} \dot{\Phi}^2 + U(\Phi) \right) \right\}. \quad (53)$$

Choosing $U(\Phi)$ to be

$$U(\Phi) = \frac{1}{2} m_s^2 \Phi^2 \quad (54)$$

and imposing the slow roll conditions *i.e*

$$\frac{1}{2}\dot{\Phi}^2 \ll U(\Phi), \quad (55)$$

the slow-roll parameters are

$$\varepsilon = \frac{m_{pl}^2}{4\pi} \left(\frac{\Omega'(\Phi)U(\Phi) + \Omega(\Phi)U'(\Phi)}{2\Omega(\Phi)U(\Phi)} \right)^2 \quad (56)$$

and

$$\eta = \frac{m_{pl}^2}{4\pi} \frac{\left[\Omega''(\Phi)U(\Phi) + \Omega(\Phi)U''(\Phi) + 2\Omega'(\Phi)U'(\Phi) \right] \Omega(\Phi)U(\Phi) + \left[\Omega'(\Phi)U(\Phi) + \Omega(\Phi)U'(\Phi) \right]^2}{2\Omega(\Phi)U(\Phi)}, \quad (57)$$

where a prime denotes differentiation with respect to Φ and $\Omega(\Phi)$ and $U(\Phi)$ are given by equations (52) and (54) respectively. Figures 9a and 9b show the value of ε and η respectively. It can be seen from these figures that the inflationary phase ends when $\Phi_f \simeq (0.11 - 0.2)M_{pl}$ where Φ_f is the value of Φ at the end of the inflation. These values of Φ lead to the number of e-folds $N \simeq 62 - 95$. Also one should note that the value of the Φ -field at the end of the inflation is below the planck scale so the η -problem [22] will not occur in this model. Figure 10 shows the value of n_s in this case. The observed value of n_s *i.e* $n_s \simeq 0.96$ occurs at $\Phi \simeq 0.16M_{pl}$ which is consistent with previous result. Figure 11 shows the tensorial spectral index. For the value of $\Phi \simeq 0.16M_{pl}$, we find from this figure that $n_T \simeq -0.2$ where lies in the acceptable range supported by observations. There exist a consistency relation between the tensorial spectral index, n_T and tensor-to-scalar ratio, r as [17]

$$n_T = -\frac{r}{8} \quad (58)$$

so we find that the tensor-to-scalar ratio in our model is $r = 0.16$. The constraint on r given in WMAP5 is $r < 0.22$ so we can see that our result is acceptable.

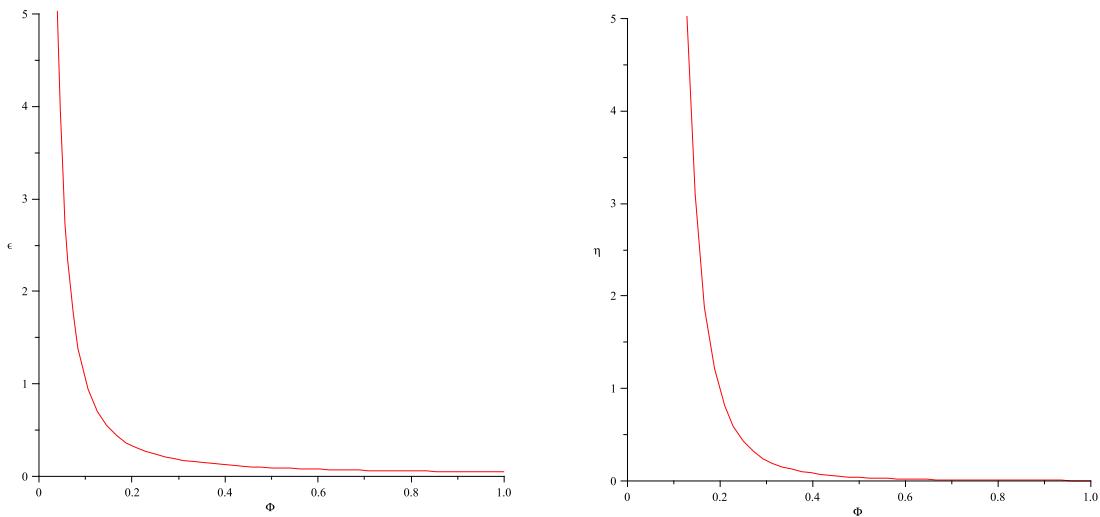


Figure 9: Graceful exit from the inflationary phase: a) ϵ versus Φ/M_{pl} . b) η versus Φ/M_{pl} .

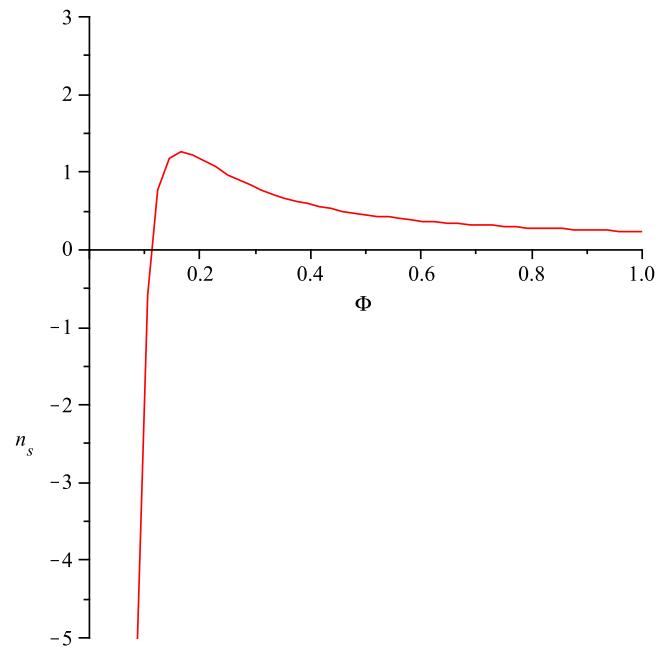


Figure 10: The value of n_s versus Φ/M_{pl}

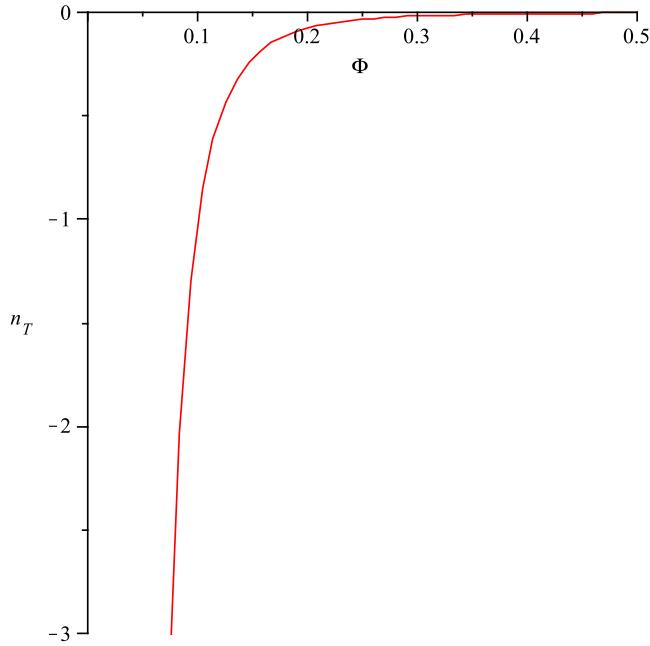


Figure 11: The value of n_T versus Φ/M_{pl}

7 Conclusion

Incorporating an inflationary phase in the Randall-Sundrum 2-brane scenario is a challenging problem because one should consider the issue of inter-brane stabilization. In this paper, we tried to construct a suitable model that matches with existing observational data of the combined WMAP5+SDSS+SNIa dataset. We did this using two different approaches: First we included the stabilization potential and Radion kinetic term directly in the Friedmann equations. The stabilization mechanism that we used is the Goldberger-Wise mechanism in which a bulk scalar field plays the role of the stabilizer. Assuming that inflation takes place in TeV brane, this model is effectively a two-field inflation scenario. One is the usual inflaton field and the other is a new field corresponding to the inter-brane separation. The inflation parameters *i.e* number of e-folds, scalar spectral index and tensor-to-scalar ratio in this model are consistent with observational data from combined WMAP5+SDSS+SNIa dataset with suitable values of the Radion field ψ . Also the value of inflaton field at the end of the inflation, ϕ_{end} is below the planck scale and therefore η problem will not occur in this setup.

The second approach is based on the assumption that the stabilization field, Φ , also plays the role of the inflaton field. Using the explicit relation for the warp factor given in reference [21], we calculated the inflation parameters in this model. Our results in this case is also consistent with observational data from WMAP5+SDSS+SNIa.

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